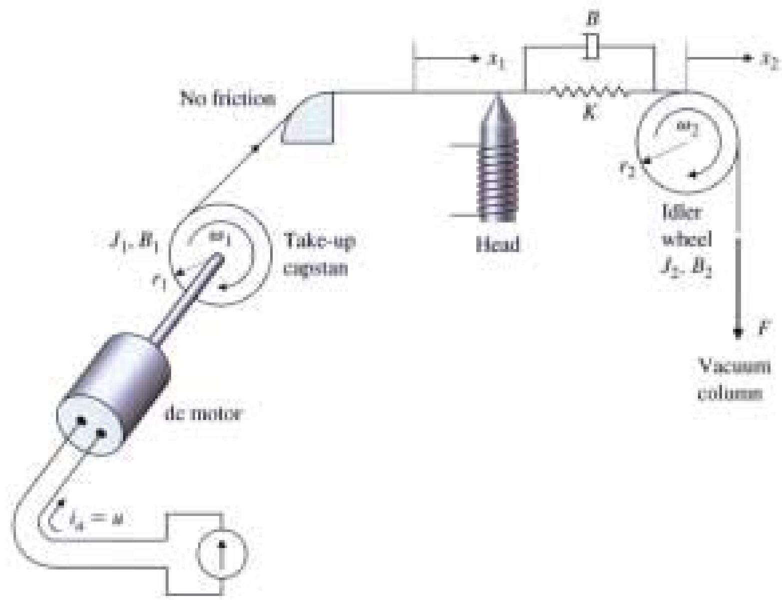
Worksheet-3

Tape drive schematic



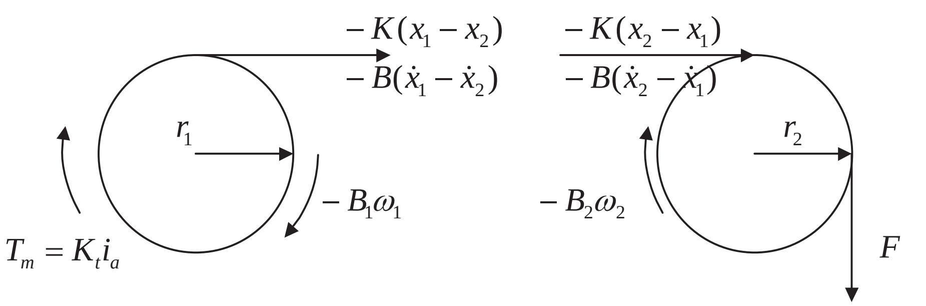
angular velocities of the two wheels are in the directions shown by the arrows.

$$
\begin{aligned}
J_1 &= 5 \times 10^{-5}\ \mathrm{kg} \cdot \mathrm{m}^2,\ \text{motor and capstan inertia} \\
B_1 &= 1 \times 10^{-2}\ \mathrm{N} \cdot \mathrm{m} \cdot \mathrm{sec},\ \text{motor damping} \\
r_1 &= 2 \times 10^{-2}\ \mathrm{m} \\
K_t &= 3 \times 10^{-2}\ \mathrm{N} \cdot \mathrm{m/A},\ \text{motor} - \text{torque constant} \\
K &= 2 \times 10^{4}\ \mathrm{N/m} \\
B &= 20\ \mathrm{N/m} \cdot \mathrm{sec} \\
r_2 &= 2 \times 10^{-2}\ \mathrm{m} \\
J_2 &= 2 \times 10^{-5}\ \mathrm{kg} \cdot \mathrm{m}^2 \\
B_2 &= 2 \times 10^{-2}\ \mathrm{N} \cdot \mathrm{m} \cdot \mathrm{sec},\ \text{viscous damping, idler} \\
F &= 6\ \mathrm{N},\ \text{constant force} \\
\dot{x}_1 &= \text{tape velocity N/ sec} \quad \text{(variable to be controlled)}
\end{aligned}
$$

(b) Write the equations in state-variable form as a set of first-order differential equations. Use the variables $(x_1, \omega_1, x_2, \omega_2, i_a)$.

(c) Use the values in part (a) and use MATLAB to find the response of x_1 to a step input in i_a.

Solution:



(a)

$$
\begin{aligned}
J_1\dot{\omega}_1 &= T_m - B_1\omega_1 - \left[B\left(\dot{x}_1 - \dot{x}_2\right) + K\left(x_1 - x_2\right)\right] r_1 \\
J_2\dot{\omega}_2 &= -B_2\omega_2 - \left[B\left(\dot{x}_2 - \dot{x}_1\right) + K\left(x_2 - x_1\right)\right] r_2 + F r_2 \\
T_m &= K_t i_a \\
\dot{x}_1 &= r_1\omega_1 \\
\dot{x}_2 &= r_2\omega_2
\end{aligned}
$$