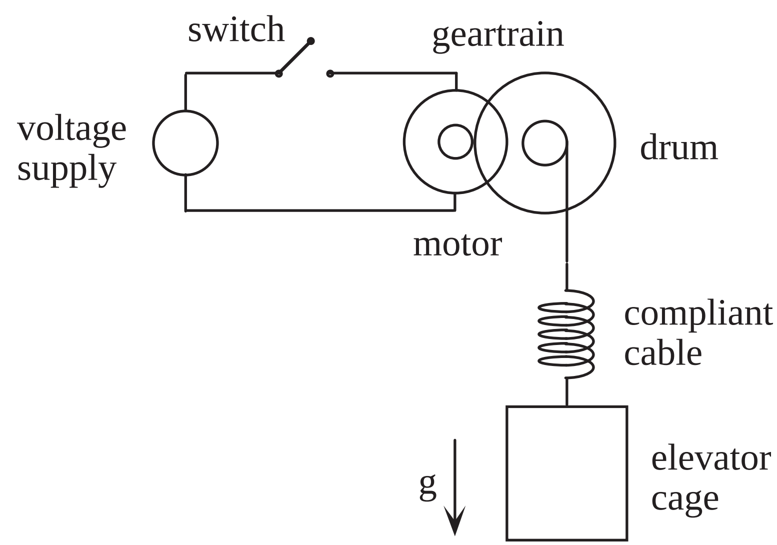
EXAMPLE: VIBRATION IN A CABLE HOIST

Problem

The cage of an elevator is hoisted by a long cable wound over a drum driven through a gear-set by an electric motor. The motor is relay-operated (i.e., either on or off) and the resulting abrupt transients cause the cage to oscillate on the hoisting cable. Because the cable has low internal friction, the oscillations persist for many cycles.



Even more important, the peak stress in the cable is almost double the steady-state stress required to support the weight of the cable.

Scenario

To solve this problem it has been proposed to introduce an electrical R-C filter between the relay and the motor terminals. The designer claims that this will smooth the transient, thereby reducing the oscillation amplitude to acceptable levels. Your task is to evaluate this proposal.

abruptly engaging the motor excites oscillation

does electrical filtering help?

Modeling goal

The simplest model competent to elucidate the effect of electrical filtering on the mechanical oscillation.

First reproduce the problem

To keep things simple assume that:

- variation of weight supported with length change may be ignored (i.e., consider small changes in elevation)
- weight is concentrated ("lumped") in the cage
- variation of cable compliance with length change may be ignored
- neglect cable internal damping (first, that emphasizes tendency to oscillation; second, it's small anyway; and third, it's easy to add later if necessary)
- drum and gear inertia may be neglected
- DC electric motor with constant magnetic field
- motor armature resistance & inductance may be neglected
- relay resistance may be neglected
- voltage supply "internal resistance" may be neglected

Direct approach

(i.e., just "write down" the differential equations)

Newtonian mechanics:

$m_{cage}\, \ddot{x}_{cage} := k_{cable}\,(x_{rim} - x_{cage}) - m_{cage}\, g$

Transmission

$\dot{x}_{rim} := r_{drum}\, \omega_{drum}$

$\omega_{drum} := n_{gear}\, \omega_{motor}$

Motor transduction characteristic:

$\omega_{motor} := e_{motor} / K_{motor}$

Switch:

$e_{motor} := e_{supply}$ if switch closed; 0 if switch open.

—a computable set of differential equations.

Given $e_{motor}(t)$ and initial conditions, $x_{cage}(t)$ is straightforward to compute. Analysis and computation is often facilitated by writing the equations in a standard state-determined form. One good choice of state variables (there are many others) yields the following.

$$\frac{d}{dt}\begin{bmatrix} x_{rim} \\ x_{cage} \\ \dot{x}_{cage} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ \frac{k_{cable}}{m_{cage}} & -\frac{k_{cable}}{m_{cage}} & 0 \end{bmatrix}\begin{bmatrix} x_{rim} \\ x_{cage} \\ \dot{x}_{cage} \end{bmatrix} + \begin{bmatrix} \frac{r_{drum}\, n_{gear}}{K_{motor}} & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} e_{motor} \\ g \end{bmatrix}$$

Laplace domain analysis:

This linear model is conveniently analyzed in the Laplace domain. Standard methods (e.g., Cramer's rule (ref. Ogata text; Rosenberg & Karnopp text)) are available to transform state-determined representations into the Laplace domain.

In this case, finding the transfer function from supply voltage to cage position by direct manipulation is straightforward.

$$(m_{cage}\, s^2 + k_{cable})\, x_{cage} = k_{cable}\, x_{rim}$$

$$s\, x_{rim} = \left(\frac{r_{drum}\, n_{gear}}{K_{motor}}\right) e_{motor}$$

$$\frac{x_{cage}}{e_{motor}}(s) = \frac{\dfrac{r_{drum}\, n_{gear}}{K_{motor}}\,\dfrac{k_{cable}}{m_{cage}}}{s\left(s^2 + \dfrac{k_{cable}}{m_{cage}}\right)}$$

A step change of motor voltage (due to switch toggling) will result in a ramp change of cage position (due to the s term in the denominator) with a superimposed sinusoidal oscillation (due to the $s^2 + k_{cable}/\, m_{cage}$ term).

A MATLAB simulation confirms this.